

DYNAMIC BALANCING OF SLIDER- CRANK MECHANISMS

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Abstract

Dynamic balance is an important feature of high speed mechanisms and robotics that need to minimize vibrations of the base. The main disadvantage of dynamic balancing, however, is that it is accompanied with a considerable increase in mass and inertia. Aiming at low-mass and low-inertia dynamic balancing a method is developed for balancing slider-crank mechanisms. Shaking force is balanced by the method of redistribution of mass and shaking moment by geared inertia counterweights. The mathematical basis for the realization of the method is static and dynamic substitution of distributed masses by concentrated point masses. The method is illustrated by two numerical examples; the results of which show that better results are produced than that of the earlier method.

Keywords— Shaking force, Shaking moment, Slider-crank mechanism.

1. INTRODUCTION

Dynamic balance, i.e., shaking force and shaking moment balance, is an important feature of machines and mechanisms that have to run at high speeds with minimum vibrations and, in addition, of free floating mechanisms such as space manipulators to maintain position and orientation. Advantages of dynamically balanced mechanisms include increased accuracy and reduced cycle times, reduced noise, wear, and fatigue [1], and improved ergonomics. Since the base of a dynamically machine does not vibrate, heavy supports and rigid floors are not needed. Balanced machines therefore can have smaller foot prints, which increase the capacity of a factory floor. Since balanced machines do not have disturbing effects to the building and surroundings [2], they also can be placed on leveled floors allowing factories to

be built vertically up. The main disadvantage of dynamic balancing is that often a considerable amount of mass and inertia is added. For moving vehicles, space manipulators, robot end effector tools, and material and transport costs, a low mass is important. For low driving torques and low driving power, low inertia is important. The methods of balancing linkages are well developed and documented in [3]. These techniques mostly are based on mass redistribution, addition of counterweights to the moving links, and attachment of rotating discs or duplication of the linkages [4].in these methods, the shaking forces and shaking moments should be minimized. For instance, counterweight balancing involves a trade off between minimizing the different dynamic reactions. Therefore determining the counterweights' mass parameters inherently constitutes an optimization problem. One of these methods is 'maximum recursive dynamic algorithm' presented by Chaudhary and Saha [5].Another method which is documented by Qi and Pennestri [6] is called 'refined algorithm'. It presents a numerically efficient technique for the optimum balancing of linkages. In this approach, instead of solving directly the dynamic equations, a technique is introduced to solve the linked dynamic equations in a "shoe string" fashion. Alici and Shirinzadeh [7] considered sensitivity analysis. In this technique they formulated the dynamic balancing as an optimization problem such that while the shaking force balancing is accomplished through analytically obtained balancing constraints, an objective based on the sensitivity analysis of shaking moment w.r.t the position, velocity, and acceleration of the links is used to minimize the shaking moment. The comprehensive mass distribution method, for an optimum balancing of the shaking force and shaking moment is used by Yu [8] to optimal balancing of the spatial RSSR mechanism. The method of linearly independent mass vectors[9] has been the most efficient method for shaking force balancing of four-bar and sixbar planar mechanisms with revolute pairs. The authors [10]-[13] used a method to balance shaking moment generated by links not directly connected to the frame. Ettefagh et al [14] described the application of Genetic algorithm for force and moment balancing of crankslider mechanism. This technique permits competing design objectives to be considered through the investigation of trade-offs between those objectives. The objective functions of the design parameters are determined and their values are minimized by adjusting the independent variables of the design and the limitation of design. The technique permits both partial force and moment balancing to be accomplished simultaneously while the desired constraints are satisfied. The forces are minimized with regard to the constraints of moments using genetic algorithm. One of the properties of genetic algorithm is binary genetic algorithm that uses the chromosomes as binary codes. Therefore binary genetic algorithm is

selected to have good trade-off between the answers accuracy and convergence speed. Van der wijk et al [15] aimed at low-mass and low-inertia dynamic balancing. The evaluation of a balanced rotatable link is found to be representative for a large group of balanced mechanisms. A rotatable link is balanced with duplicate mechanisms, with a counter mass and a separate counter rotation and with a counter-rotary counter mass. The equations for the total mass and the inertia are derived and compared analytically while the balancing principles are compared numerically. The results showed that the duplicate mechanism balanced link is the best compromise for low-mass and low-inertia but requires a considerable space. For the counter rotary counter mass balanced link and the separate counter rotation balanced link that are more compact, there is a trade-off between mass and inertia for which the counter-rotary counter mass-balanced link is the better of the two. Ilia, D. and Sinatra [16] derived design equations and techniques for the dynamic balancing of five-bar linkage, using a novel and simplified approach. In order to derive the equations of the mechanism the natural orthogonal complement method is used. Subsequently an optimization method for the dynamic balancing of the five-bar linkage is proposed. The conditions of dynamic balancing of the five-bar linkage are expressed as seven equations and four equalities, with twelve linkage parameters. The dynamic balancing of the mechanism is formulated and solved as an optimization problem under equality constraints. Cheng-Ho LI and Pei-Lum TSO [17] proposed a concept of using both a linkage balancer and counterweight disks to reduce shaking force and shaking moment of high speed mechanical presses. The linkage balancer is designed with analytic synthesis method for matching critical shaking forces at the right timing. Counterweight disks positions and masses are designed with an optimum method that considers minimizing the magnitude and the fluctuation of the shaking effect. Comparatively, the linkage balancer and the counterweight disks are apt at dealing with shaking force and shaking moment respectively.

The present paper is the extension work of authors [10]-[13]. The results obtained are nearly 90 times better than the previous method results. The paper is organized as follows: section 1 deals with introduction, section 2 presents articulation dyad. Dynamic balancing of slider-crank mechanism is given in section 3.Numerical examples and results are discussed in section 4.Conclusions are given in section 5.

2. ARTICULATION DYAD

A. COMPLETE SHAKING FORC"E AND SHAKING MOMENT BALANCING OF AN ARTICULATION DYAD:

An open kinematic chain of two binary links and one joint is called a dyad. When two links are articulated by a joint so that movement is possible that arrangement of links is known as articulation dyad.

The familiar scheme of complete shaking force and shaking moment balancing of an articulation dyad [10]-[13] is shown in Fig.1.

To link 2 is added a counterweight which permits the displacement of the center of mass of link 2 to joint A. then, by means of a counter weight with mass m_{cw_1} [Fig.1] a complete balancing of shaking force is achieved. A complete shaking moment balance is realized through four gear inertia counter weights 3-6, one of them being of the planetary type and mounted on link 2.



Fig.1. Complete shaking force and shaking moment balancing of an articulation dyad

B. COMPLETE SHAKING FORCE AND SHAKING MOMENT BALANCING OF AN ARTICULATION DYAD BY GEAR INERTIA COUNTERWEIGHTS MOUNTED ON THE BASE:

The scheme used in the present work [Fig.2] is distinguished from the earlier scheme by the fact that gear 3 is mounted on the base and is linked kinematically with link 2 through link1'.



Fig.2 Complete shaking force and shaking moment balancing of an articulation dyad by gear inertia counterweights mounted on the base

The link 1' is connected to link 2 at midpoint of link 2.

To prove the advantages of such a balancing, the application of the new system with the mass of link 1' not taken into account is considered. In this case (compared to the usual method Fig.1), the mass of the counter weight of link 1 will be reduced by an amount

$$\delta m_{cw_1} = \frac{m_3 l_{OA}}{r_{cw_1}}$$
(1)

where,

 m_3 is the mass of gear 3, l_{OA} is the distance between the centers of hinges O and A, r_{cw_1} is the rotation radius of the center of mass of the counter weight.

It is obvious that the moment of inertia of the links is correspondingly reduced. If the gear inertias are made in the form of heavy rims in order to obtain a large moment of inertia, the moments of inertia of the gear inertia counter weights may be presented as

$$I = \frac{m_i D_i^2}{4} (i=3...6).$$

Consequently, the mass of gear 6 will be reduced by an amount

$$\delta m_{_{6}} = 4(m_{_{3}}l_{_{OA}}^{^{2}} + \delta m_{_{CM}}r_{_{CM}}^{^{2}})\frac{T_{_{6}}}{D_{_{6}}^{^{2}}T_{_{5}}}$$
(2)

Where,

 T_5 and T_6 are the numbers of teeth of the corresponding gears. Thus, the total mass of the system will be reduced by an amount

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$$\delta m = \delta m_{cw_1} + \delta m_6 \tag{3}$$

Here the complete shaking force and shaking moment balancing of the articulation dyad with the mass and inertia of link 1' taken into account is considered. For this purpose initially, statically replace mass m'_1 of link 1' by two point masses m_B and m_c at the centers of the hinges B and C

$$m_B = m_{1'} l_{CS_{1'}} / l_{BC}$$

$$m_C = m_{1'} l_{BS_{1'}} / l_{BC}$$
(4)

Where, l_{BC} is the length of link 1, $l_{CS_{1'}}$ and $l_{BS'_{1}}$ are the distances between the centers of joints C and B and the center of mass S'_{1} of link 1', respectively.

After such an arrangement of masses the moment of inertia of link 1' will be equal to

$$I_{S_1'}^* = I_{S_1'} - m_{1'} l_{BS_1'} l_{CS_1'}$$
(5)

where,

 $I_{S'_1}$ is the moment of inertia of link 1' about the center of mass S'_1 of the link. Thus a new dynamic model of the system is obtained, where the link 1' is represented by two point masses m_B, m_C and has a moment of inertia $I_{S'_1}^*$.

This fact allows for an easy determination of the parameters of the balancing elements as follows:

$$m_{CW_2} = (m_2 l_{AS_2} + m_B l_{AB}) / r_{CW_2}$$
(6)
where

where, m_2 is the mass of link 2, l_{AB} is the distance between the centers of the hinges A and B, l_{AS_2} is the distance of the center of hinge A from the center mass of S_2 of link 2, r_{CW_2} is the rotation radius of the center of mass of the counterweight with respect to A, and

$$m_{CW_1} = \left[(m_2 + m_{CW_2} + m_B) l_{OA} + m_1 l_{OS_1} \right] / r_{CW_1}$$
(7)

where, m_1 is the mass of link 1, l_{OS_1} is the distance of the joint center O from the center of mass S_1 of link 1.

$$m_{CW_3} = m_C l_{OC} / r_{CW_3}$$
(8)

where,

 $l_{OC} = l_{AB}$, r_{CW_3} is the rotation radius of the center of mass of the counterweight. Taking into account the mass of link 1' brings about the correction in Eq.(3) in this case,

$$\delta m = \delta m_{CW_1} + \delta m_6 - \delta m_1' \tag{9}$$

where,

 $\delta m'_1$ is the value deciding the change in the distribution of the masses of the system links resulting from the addition of link1'.

3. DYNAMIC BALANCING OF SLIDER-CRANK MECHANISM

The slider-crank mechanism is a modification of the basic four-bar mechanism. In the basic four-bar mechanism if one of the turning pairs is replaced by a sliding pair then the mechanism obtained is called as Slider-crank mechanism. The slider-crank mechanisms are usually found in Reciprocating steam, Rotary internal engines, Pendulum pump or bull engines, Oscillating cylinder engines, Rotary internal combustion engines, Crank-slotted lever quick return motion mechanisms and Whit-worth quick return motion mechanisms etc. In the slider-crank mechanism shown in Fig.3, link 2 is crank, link 3 is connecting rod, link 4 slider and link is the fixed one. When the slider-crank mechanism runs at high speeds shaking forces and shaking moments are developed in the mechanism, these undesirable qualities of the mechanism are to be eliminated; the balanced slider-crank mechanism is shown in Fig.4



Fig.3 Slider-crank mechanism

A. SHAKING FORCE BALANCING OF THE MECHANISM

For link 3 to be dynamically replaced by two point masses m_{B3} and m_{P3} the condition to be satisfied is $k_3^2 = l_{P_3S_3} l_{BS_3}$; where l_{BS_3} is arbitrarily chosen and $l_{P_3S_3}$ is obtained from the above condition

$$m_{B3} = \frac{m_3 l_{P_3 S_3}}{(l_{P_3 S_3} + l_{BS_3})}$$
$$m_{P3} = \frac{m_3 l_{BS_3}}{(l_{P_3 S_3} + l_{BS_3})}$$



Fig.4 Balanced Slider-crank mechanism

After link 3 is dynamically replaced by two point masses it is kinematically connected to its corresponding gear inertia counterweight 6 by link 2', moreover link 2' is statically replaced by two point masses m_D and m_C .

$$m_{c} = \frac{m_{2} l_{os_{2}} / l_{oA}}{m_{c}} = \frac{m_{2} l_{os_{2}} / l_{oA}}{(l_{os_{2}} + l_{cs_{2}})}; m_{D} = \frac{m_{2} l_{cs_{2}} / (l_{os_{2}} + l_{cs_{2}})}{(l_{os_{2}} + l_{cs_{2}})}$$

Counterweight against link 3 can be obtained as

$$m_{cw_{\rm B}} = (m_3 l_{AS_{\rm B}} + m_4 l_{AB} + m_c l_{AC}) / r_{cw_{\rm B}}$$
(10)

$$r_{cw_s} = l_{p_s s_s} - l_{AS_s}$$

Where $r_{cw_{B}}$ = radius of rotation of counterweight $m_{cw_{B}}$

Link 2 is dynamically replaced by two point masses m_{A2} and m_{P2} using the condition $k_2^2 = l_{P_2S_2} l_{AS_2}$; where l_{AS_2} is arbitrarily chosen and $l_{P_2S_2}$ is obtained from the above condition

$$m_{A2} = m_2 l_{p_2 s_2} / (l_{p_2 s_2} + l_{AS_2})$$

$$m_{p2} = m_2 l_{AS_2} / (l_{p_2 s_2} + l_{AS_2})$$

$$k_2^2 = l_{p_2 s_2} l_{AS_2}$$
Counterweight against link 2 can be obtained as
$$m_{cw_2} = \frac{m_2 l_{OS_2} + (m_{cw_2} + m_c + m_3 + m_4) l_{OA}}{r_{cw_2}}$$
(11)
Where $r_{cw_2} = l_{p_2 s_2} - l_{OS_2}$
Linear acceleration at points A, B, is
$$A_{A} = ia\alpha_2 e^{i\theta_2} - a\omega_2^2 e^{i\theta_2}$$

$$A_{BA} = ib\alpha_3 e^{i\theta_3} - b\omega_3^2 e^{i\theta_3}$$

$$= -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\omega_3^2 \cos\theta_3 + A_B b \alpha_3 \sin\theta_3$$
Total shaking force generated in the mechanism is a

Total shaking force generated in the mechanism is given by $F = -(m_2 (A_G)_2 + m_3 (A_G)_3 + m_4 (A_G)_4)$ (12)

B. SHAKING MOMENT BALANCING OF THE MECHANISM

The shaking moment generated by the mechanism is determined by the sum

$$M^{\text{int}} = M_{2}^{\text{int}} + M_{3}^{\text{int}}$$
(13)
Where,

$$M_{2}^{int} = (I_{S_{2}} + m_{2}l_{OS_{2}}^{2} + (m_{c} + m_{cw_{2}} + m_{3} + m_{4})l_{OA}^{2} + m_{cw_{2}}r_{cw_{2}}^{2} + I_{S_{2}'}^{*} + m_{2}'l_{DS_{2}'}^{2})\alpha_{2}$$

 $M_3^{\rm int} = 2m_D l_{OD}^2 \alpha_3$

 M_2^{int} , M_3^{int} are the Shaking moments generated by links 2 and 3 respectively

 I_{S_2} is the mass moments of inertia of link 2

 $I_{S_2}^*$ is the changed moment of inertia of links 2'.

 $\alpha_{2,\alpha_{3}}$ are the angular accelerations of link 2 and 3 respectively.

Shaking force of the mechanism by the Proposed method:

$$F_{\text{Proposed}} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_2 A_{G2})$$

Shaking moment of the mechanism by the Proposed method:

$$M_{proposed}^{\text{int}} = M_2^{\text{int}} + M_3^{\text{int}}$$

Shaking force of the mechanism by Gao Feng's method:

$$F_{Gaofeng} = -(m_2 A_{G2} + m_3 A_{G3} + m_4 A_{G4} + m_{G6} A_{G6})$$

Shaking moment of the mechanism by Gao Feng's method:

$$M_{Gaofeng}^{int} = M_2^{int} + (I_{s6} + 2m_{G6}l_{OA}^2)\alpha_2$$

4. NUMERICAL EXAMPLES

NUMERICAL EXAMPLE I:

The slider-crank mechanism shown in fig.3 has the following parameters: Here the connecting rod length is assumed to be almost 3 times the crank length. The parameters for the slider-crank mechanism are assumed as $l_{oA} = 1.4m$, $l_{AB} = 4m$, $l_{oB} = 1m$, $m_2 = m_2^2 = 2.5kg$, $m_3 = 3.5kg$, $m_4 = 5kg$, follows: $k_2 = 1.5m$, $k_3 = 2.5m$, $l_{AS3} = l_{BS3} = 2m$, $l_{oS2} = 0.7m$, $l_{AC} = 0.3m$,

$$l_{max} = l_{max} = 0.7m, \theta_{2} = 30^{\circ}, \omega_{2} = 200 rad / s, \alpha_{2} = 10 rad / s^{2}$$

A. COMPARISON BETWEEN THE RESULTS OF GAO FENG'S METHOD AND THE

PROPOSED METHOD

The results of shaking forces of numerical example I of slider-crank mechanism are given in table 1.The results show that at all angular positions of the crank shaking forces in mechanism by proposed method are far less than that of by Gao Feng's method. The results of shaking moments generated by numerical example I of the mechanism are given in table 2.The shaking moments at various angular positions of the crank show that proposed method values are less than that of by Gao Feng's method.

Shaking force in Gao Feng's method is maximum, 115.84×10^5 N, at 240^0 and minimum, 108.56×10^5 N, at 360^0 . Shaking force in Gao Feng's method gradually increases from 0^0 to 120^0 , and again gradually decreases from 240^0 to 360^0 of crank angle. Shaking moment in Gao Feng's method is constant, 14.4920×10^3 N-m, from 0^0 to 360^0 of crank angle. For slider-crank mechanism in the proposed method shaking force is maximum, 4.5236×10^5 N, at 240^0 and minimum, 0.3277×10^5 N, at 60^0 . Shaking force gradually increases from 60^0 to

 240° . Shaking moment of slider-crank mechanism in the proposed method is maximum, 4.9549×10^{3} N-m, at 90° and minimum, 0.5220×10^{3} N-m, at 330° . Shaking moment of slider-crank mechanism gradually decreases from 90° to 330° .

TABLE 1 SHAKING FORCE COMPARISON OF NUMERICAL EXAMPLE-I OF SLIDER-CRANK MECHANISM

Crank	Shaking	Shaking
angle	force	force
(deg)	generated	generated
	in	in Gao
	proposed	Feng's
	method	method 10 ⁵
	$10^{5} \mathrm{N}$	N
0	-2.7481	108.5600
30	-1.7570	109.5500
60	0.3277	111.6400
90	2.2656	113.5800
120	3.1281	114.4400
150	3.0931	114.4100
180	2.8519	114.1600
210	3.4927	114.8000
240	4.5236	115.8400
270	4.3257	115.6400
300	1.7244	113.0400
330	-1.3560	109.9600
360	-2.7481	108.5600

TABLE 2 SHAKING MOMENT COMPARISON OF NUMERICAL EXAMPLE-I OF SLIDER-CRANK MECHANISM

Crank	Shaking	Shaking
angle	moment	moment
(deg)	generated	generated in
	in	Gao Feng's
	proposed	method 10^3
	method	N-m
	10^{3} N-m	
0	2.0918	14.4920
30	3.4303	14.4920
60	4.5057	14.4920
90	4.9549	14.4920
120	4.5065	14.4920
150	3.4317	14.4920
180	2.0935	14.4920
210	0.5236	14.4920
240	-1.2213	14.4920
270	-2.1485	14.4920
300	-1.2223	14.4920
330	0.5220	14.4920
360	2.0918	14.4920

The shaking force and shaking moment values by the proposed method are less than that of by Gao Feng's method. The shaking forces and shaking moments calculated at every 30^{0} of crank angle for the slider-crank mechanism are almost less at every interval by proposed method. It can be concluded from shaking forces and shaking moments of slider-crank mechanisms that better results are obtained by proposed method over Gao Feng's method.

NUMERICAL EXAMPLE II:

Here the connecting rod length is assumed to be 4 times the crank length. The slider-crank mechanism shown in fig.3 has the following parameters. $l_{oA} = 1m$, $l_{AB} = 4m$, $l_{oB} = 1.4m$, $\omega_2 = 200 rad / s$, $\alpha_2 = 10 rad / s^2$

B. COMPARISON BETWEEN THE RESULTS OF GAO FENG'S AND THE PROPOSED METHODS

TABLE 3 SHAKING FORCE COMPARISON OF NUMERICAL EXAMPLE-II OF SLIDER-CRANK MECHANISM

Crank	Shaking	Shaking
angle	force	force
(deg)	generated in	generated in
	proposed	Gao Feng's
	method 10^5	method 10^5
	Ν	Ν
0	-1.5186	109.7900

30	-0.9760	110.3400
60	0.2555	111.5700
90	1.5020	112.8100
120	2.2557	113.5700
150	2.4882	113.8000
180	2.4813	113.7900
210	3.0305	114.3400
240	3.6590	114.9700
270	3.3755	114.6900
300	1.6596	112.9700
330	-0.4327	110.8800
360	-1.5186	109.7900

TABLE 4 SHAKING MOMENT COMPARISON OF NUMERICAL EXAMPLE-II OF SLIDER-CRANK MECHANISM

Crank	Shaking	Shaking
angle	moment	moment
(deg)	generated	generated in
	in	Gao Feng's
	proposed	method 10^3
	method	N-m
	10^{3} N-m	
0	2.0279	14.4920
30	3.0457	14.4920
60	3.7742	14.4920
90	4.0503	14.4920
120	3.7747	14.4920
150	3.0467	14.4920
180	2.0291	14.4920
210	0.8051	14.4920
240	-0.4328	14.4920
270	-1.0235	14.4920
300	-0.4335	14.4920
330	0.8040	14.4920
360	2.0279	14.4920

The results of shaking forces of the mechanism of numerical example II are shown in table 3. The shaking forces of the mechanism by proposed method are far less than that of by Gao Feng's method i.e, almost 90 times less than Gao Feng method values. The shaking moment results are shown in table 4. From the results it can be observed that the shaking moments by proposed method are less than that of by Gao Feng's method.

5. CONCLUSIONS

Shaking force is balanced by the method of redistribution of mass and shaking moment by geared inertia counterweights. When compared the values of shaking forces and shaking

moments for both numerical examples, shaking forces and shaking moments by proposed method are less than that of Gao Feng's method. Hence it is proved that the proposed method has produced better results compared to Gao Feng's method. The balanced mechanism is constructively more efficient and occupies less space.

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