Scholarly Research Journal for Humanity Science & English Language
Online ISSN 2348-3083, SJ IMPACT FACTOR 2024: 8.058
https://www.srjis.com/issues_data/226
Peer Reviewed, Refereed & Indexed Journal Dec-Jan 2024, Vol-12/61

A COMPARATIVE STUDY OF AGE OF INFORMATION OF THE QUEUEING / COMMUNICATION SYSTEMS E_K / M / 1 AND M / E_K / 1

Satish Chandra Tiwari
Associate Professor, Retd. Dept. of Mathematics, S.V. College, Aligarh

Sanjay Chaudhary
Prof. & Head Dept. of Mathematics, Institute of Basic Sciences, Khandhari, Agra
Dr. B. R. Ambedkar University, Agra

Paper Received On 20 December 2023

Peer Reviewed On 25 January 2024

Published On: 01 February 2024

Abstract

In this research paper the Age of Information of the queueing / communication system E_k / M / 1 and its dual M / E_k / 1 has been calculated, compared and discussed with the variation of phases (k) and the traffic intensity (ρ). The study reveals that the age of information decreases monotonically as k increases and is a function of ρ.

Keywords: *Age of Information, Communication System, Traffic Intensity.*

Introduction

Age of Information (AOI) refers to a metric that measures 'freshness' of the information available. More precisely, it gives the status update in terms of the average time since the generation of the message until it is received at the destination.

If the freshness of a message or packet generated by a source at time u (t) is received at a destination at time t, then age is given as

A (t) = t – u (t)

And the time average age over a time interval (0, T) is expressed as,

$$A(T) = \frac{1}{T}\int_0^T [t - u(t)]dt$$

or $$A(T) = \frac{1}{T}\int_0^T A(t)dt$$

The age of status up date in steady state is defined as,

$$A = \lim_{T\to\infty} A(T) = \lim_{T\to\infty}\int_0^T A(t)dt$$

Peak age of information, refers to the sum of the update interval and response time of the message / packet i.e. it is the sum of the inter – arrival and the system time of the packet or message.

Formulation of average age of information and peak age of information: In the study, A_{av} and A_{peak} will be used to denote average and peak age, respectively.

Mathematically A_{av} and A_{peak} are defined as:

$$A_{av} = \lambda E[IT + \frac{I^2}{2}],$$ kaul et al. (2012)

$$= \lambda \left\{ E[IT] + E[\frac{I^2}{2}] \right\}$$ & --------------------(i)

A_{peak} = E [I + T], Longbo and Modiano (2015) --------------------(ii)

Where,

I, T are the inter-arrival and system times respectively corresponding to a packet.

If W, S be the time spent by a packet in queue and in service respectively then,

T = W + S,

If λ, μ be the arrival and service rates then,

$$\lambda = \frac{1}{E[I]}, \mu = \frac{1}{E[S]}$$

On incorporating all these results we have

$$A_{av} = \lambda \{E [WI] + E [S] E [I] + E [\frac{I^2}{2}]\}$$ --------------------(iii)

& A_{peak} = E [I] + E [S] + E [W] --------------------(iv)

“Longbo and Modiano (2015) proved by a lemma that A_{peak} approximates AoI for single class general queueing system – G / G / 1”

Also,

$$A_{av} \le W + \frac{1 + C_a^2}{2}.E[I]$$ Ref 8 (inequality 42) --------------------(v)

Ca, is the coefficient of variation of inter-arrival distribution.

In the light of the research studies (Ref : 1, 2, 8)

We can approximate AoI, For the queueing systems M/ E_k / 1 and E_k / M / 1 as follows:

$$\text{AoI}(M/E_k/1) = \frac{1}{\mu}\left[1+\frac{1}{\rho}+\frac{k+1}{2k}\frac{\rho}{1-\rho}\right] \quad \& \qquad \text{--------------------(vi)}$$

$$\text{AoI}(E_k/M/1) = \frac{1}{\mu}\left[1+\frac{k+1}{2k\rho}+\frac{\alpha}{1-\alpha}\right] \qquad \text{--------------------(vii)}$$

Where $\alpha(0 \le \alpha \le 1)$, is the least, real root of the equation,

$$z = \left(\frac{k\lambda}{k\lambda+\mu-z}\right)^k$$

With a little algebraic exercise we have,

for $k = 1$, $\alpha = \rho$

& for $k = \infty$, α is given by –

$$\alpha = e^{-(1-\alpha)/\rho}$$

History / Literature Review: The concept of AoI was first coined in 2012. Since then, several studies have been conducted to formulate and study this concept. The concept has attracted the attention of network users in varied fields where the age of information is a matter of concern for system optimization. A brief review of the related work is as follows:

Kaul et al. (2012). Formulated AoI for the queueing systems M/M/1, M/D/1 and D/M/1. They also calculated the optimal values of utilization for each of these systems to minimize the age of information. **Yates and Kaul (2012)** discussed the age of information for the queueing system with multiple sources and derived significant results. **Kaul and Yates (2012)** looked at the status updates through queues having the discipline FCFS and dealt with the queues having queueing discipline FCFS. **Costa et al. (2014)** discussed the age of information with packet management. **Costa et al. (2014)** gained insight into the age of information and determined the metric for the queueing system M/M/1/1, M/M/1/2/, M/M/1/2*. In the queueing system M/M/1/2* the packet in waiting is replaced by a new arrival. **Longbo and Modiano (2015)** derived the expressions for peak age of information for the general queueing system G/G/1. In their study, they used a lemma to show that peak AoI approximates AoI for the general queueing systems G/G/1. **Inoue et al. (2019)** dealt with single server queues and explored general results for age of information. **Yates et al. (2021)** delved into the concept of age of information by going through a critical survey of the previous related work. In their study, they explored the expressions for AoI of different queueing systems. They also discussed the age of queueing networks. In the last section of their research paper, they discussed applications of age in games, learning, caching and protocols.

Table – 1 (For μ = 1, ρ = λ = .1)

Number of phases, k	AoI ($M/E_k/1$) $\left[\frac{1}{\mu}\left(1+\frac{1}{\rho}+\frac{k+1}{2k}\frac{\rho}{1-\rho}\right)\right]$	AoI ($E_k/M/1$) $\left[\frac{1}{\mu}\left(1+\frac{k+1}{2k\rho}+\frac{\alpha}{1-\alpha}\right)\right]$
1	11.1111	11.1111
2	11.0833	8.5300
3	11.0740	7.6795
4	11.0694	7.2568
5	11.0667	7.0041
6	11.0648	6.8361
7	11.0635	6.7162
8	11.0625	6.6265
9	11.0617	6.5567
10	11.0611	6.5010
.	.	.
..	.	.
$k = \infty$	11.0555	$6 + \alpha$*

*** $0 \le \alpha \le 1$ is an admissible root of the equation $\alpha = e^{-(1-\alpha)/\rho}$**

Table – 2 (For μ = 1, ρ = λ = .5)

Number of phases, k	AoI ($M/E_k/1$) $\left[\frac{1}{\mu}\left(1+\frac{1}{\rho}+\frac{k+1}{2k}\frac{\rho}{1-\rho}\right)\right]$	AoI ($E_k/M/1$) $\left[\frac{1}{\mu}\left(1+\frac{k+1}{2k\rho}+\frac{\alpha}{1-\alpha}\right)\right]$
1	4	4
2	3.7500	3.1180
3	3.6667	2.8270
4	3.6250	2.6825
5	3.6000	2.5962
6	3.5833	2.5388
7	3.5714	2.4979
8	3.5625	2.4674
9	3.5555	2.4436
10	3.5500	2.4246
.	.	.
.	.	.
$k = \infty$	3.5000	$2 + \alpha$*

*** $0 \le \alpha \le 1$ is an admissible root of the equation $\alpha = e^{-(1-\alpha)/\rho}$**

Table – 3 (For μ = 1, ρ = λ = .9)

Number of phases, k	**AoI ($M/E_k/1$)** $\left[\frac{1}{\mu}\left(1+\frac{1}{\rho}+\frac{k+1}{2k}\frac{\rho}{1-\rho}\right)\right]$	**AoI ($E_k/M/1$)** $\left[\frac{1}{\mu}\left(1+\frac{k+1}{2k\rho}+\frac{\alpha}{1-\alpha}\right)\right]$
1	11.1111	11.1111
2	8.8611	8.4221
3	8.1111	7.5254
4	7.7361	7.0774
5	7.5111	6.8086
6	7.3611	6.6295
7	7.2539	6.5016
8	7.1736	6.4057
9	7.1111	6.3310
10	7.0611	6.2714
.	.	.
.	.	.
$k = \infty$	6.6111	$1.5555 + \alpha^*$

*** $0 \le \alpha \le 1$ is an admissible root of the equation $\alpha = e^{-(1-\alpha)/\rho}$**

Concluding discussion: From the tables, it is evident that:

(i) In both systems, the age of information decreases monotonically with k.

(ii) Average age of information for E_k / M/<, M/E_k / 1

(iii) The Age of information is a function of k, ρ and for a given k can be optimized with an optical value of ρ.

(iv) ρ is optimal for .51 < .63 approximately

(v) For k = 1 both the systems are identical to the system M/M/1

(vi) For k = ∞, these systems provide AoI for D/M/1 and M/D/1 systems.

(vii) There is rapid variation with k, in AoI for the system E_k/M/1 in comparison to M/E_k/1

(viii) Expressions for A_{av} and A_{peak} show that

$A_{av} \le A_{Peak}$ for $C_a \le 1$ & $A_{av} > A_{Peak}$ for $C_a > 1$ and E[s] = 0; Where C_a is the coefficient of variation of the inter-arrival distribution.

(ix) The Age of information can be optimized by taking appropriate values of k and ρ.

Applications / Significance: Age of information is an important and significant metric to measure the freshness of status updates. It is very useful where the optimization of a communication system needs freshness of the update, e.g., for the players in the games, machine learning, load balancing, mobile networks, to quote but a few.

References:-

Costa, M. et al.: "Age of Information with Packet Management," Proceedings of ISIT, 2014

Costa, M. et al.: "On the Age of Information in Status Update Systems with Packet Management" IEEE, ISTT 2014.

Inoue, Yoshiaki et al. : "A General Formula for the Stationary Distribution of the Age of Information and Its Application to Single Server Queues," arXiv : 1804.06139V2, 2019.

Kaul K. Sanjeet et al..: "Status Updates Through Queues" 978-1-4673-3140-1/12 @ 2012 IEEE.

Kaul, Sanjeet et al.: "Real–Time Status : How Often Should One Update?" In Proc. IEEE, International Conference on Computer Communications (INFOCOM), 2012.

Longbo Huang and Eytan Modiano, : "Optimizing Age-of-Information in a Multiclass Queueing System," arXiv 1504.05103, April, 2015.

Yates D. Roy and Kaul S.K. : "The Age of Information: Real-Time Status Updating by Multiple Sources," IEEE ISTI 2012.

Yates, D. Roy et al.: "Age of Information: An Introduction and Survey," IEEE Journal of Selected Areas in Communications 39(5), 1183-1210, 2021